# BASICS OF BXM FUTURES

BXM futures are based on the CBOE S&P 500 BuyWrite Index (BXM), CBOE's widely followed BuyWrite index. In essence, the BXM is a perpetual covered call on the S&P 500, with the call sold at-the-money. An investment in the BXM portfolio provides a leveraged position on the total return of the S&P 500 with a cap at the S&P 500 call strike. BXM futures in turn provide a leveraged exposure to the BXM, and therefore a double-leveraged exposure to the capped total return of the S&P 500.

Day-to-day, and from listing to expiration, the performance of BXM futures is expected to closely track the performance of the BXM. This is because BXM futures are linked to the BXM by an arbitrage relationship similar to the "carry" arbitrage that ties stock index futures and the underlying stock index. Hence a key factor in forecasting the return and volatility of a BXM futures position is the BXM itself.

## BXM Revisited

The BXM measures the performance of a portfolio long the S&P 500 index and short a one-month at-the-money call on the S&P 500. Shortly after the S&P 500 call expires on the third Friday of the month, a new call is sold, a process referred to as the roll. The strike of the call is at-the-money or as close as possible above if there is no listed strike exactly at-the-money.

Calculation

The BXM is calculated by chaining the successive daily rates of return of this portfolio since June 1, 1988 when the BXM's value was set at 100. The real-time calculation presumes that the dividends paid by the S&P 500 portfolio are reinvested daily at the open in the covered call position.

On non-roll dates, the daily rate of return used to calculate the BXM is the ratio of the closing value of the BXM portfolio (S&P 500 + dividends – call price) and of its value at the previous close (S&P 500 – call price). On roll dates, the daily rate of return used to calculate the BXM is compounded from three rates of return:

(1) The rate from the previous close to the Special Opening Quotation (SOQ) of the S&P 500; the SOQ is typically determined shortly after the market open but there can delays if some S&P 500 stocks take time to open because of market congestion,

(2) The rate from the SOQ to 12:00 p.m. Eastern time (ET), and

(3) The rate from 12:00 p.m. ET to the close of the day. The new call is deemed sold at the volume-weighted-average price (VWAP) from 11:30 a.m. to 12:00 p.m. ET; the S&P 500 is also deemed bought at its volume-weighted average value (VWAV) during the same period. The VWAV of the S&P 500 therefore enters the second and third rates of return compounded in the roll-date return and the call VWAP enters the

third rate of return. The equations that govern the rate of return are at **http://www.cboe.com/micro/bxm/BXMDescription-Methodology.pdf .**

Replication

To replicate the rate of return on a non-roll date, an investor re-invests daily distributions of S&P 500 dividends in the BXM portfolio at the open. To approximate the rate of return on a roll date, an investor (1) lets the S&P 500 call expire, (2) re-invests the roll-date dividends in the S&P 500 portfolio, and, if the call has expired in the money, finances the settlement by liquidating the appropriate fraction of the S&P 500 portfolio. (3) carries the resulting uncovered S&P 500 portfolio until 11:30 a.m ET, (4) sells the call at the VWAP from 11:30 a.m. to 12:00 p.m., and (5) carries the rebalanced long S&P 500 / short S&P 500 call portfolio to the close.

## Settlement of BXM Futures

BXM futures settle on the roll date to a special opening value of the BXM ($BXM_{SOQ}$) calculated as the product of the value of the BXM at the previous close and the rate of return of the BXM from the previous close to the joint expiration of the S&P 500 and S&P 500 call. The settlement value of the BXM futures price is:

$$\mathbf{F_T = BXM_{SOQ} = BXM_{T-1} * (1+R_a)}.$$

where $1+ R_a = (SOQ + D_T - C_{settle}) / (S\&P500_{T-1} - C_{T-1})$, and $C_{settle}$ is the settlement value of the call.

The dollar return to an investor who buys approximately one contract at time t and holds futures to expiration is **$100 * ($F_T - F_t$)**. The nitty-gritty behind the "approximately" is that the number of contracts held daily must be discounted by an interest rate factor to compensate for the marking-to-market of the futures and that interest rates are assumed to be non-random.

## Arbitrage between BXM and BXM futures

Similar to stock index futures, there is an arbitrage between BXM futures and the BXM. This arbitrage ensures that the futures closely track the BXM. The arbitrage pairs a leveraged purchase of the BXM with a short position in BXM futures, or conversely, a short position in the BXM and a long position in BXM futures. The long BXM position is financed to the futures expiration date at the relevant money market rate, and it is managed to mimic the daily BXM rate return, on roll as well as non-roll dates. As in any arbitrage, execution slippage and transaction costs need to be accounted for. With these caveats, the arbitrage relationship should still hold and lead to the following relationship:

$$\mathbf{F_t = BXM_t * (1+r_{tT})},$$

where $F_t$ is the futures price at date t, and $r_{tT}$ is a money market rate from date t to the expiration date T of the futures. Note that this is slightly different from the arbitrage relationship between stock index futures and the underlying stock index where the dividends must be netted from the price of the futures.

An important implication of the arbitrage between BXM futures and the BXM is that the rate of change of BXM futures is proportional to the rate of return of the BXM, and similarly the daily dollar variation of BXM futures is approximately[1[1]] proportional to the daily variation of the BXM:

$$\frac{F_t}{F_{t-1}} = (1+R_t) * (1+\Delta r_{tT}), \text{ and}$$

$$\mathbf{\$100 * (F_t - F_{t-1}) \approx \$100 * (1+r_{t-1T}) (BXM_t - BXM_{t-1})}$$

where $R_t$ is the rate of return of the BXM and $\Delta r_{tT}$ is the rate of change of the money market rate. The money market rate converges to 0 as the futures approach expiration.

The carry arbitrage between BXM futures and the BXM immediately implies the following calendar arbitrage between the prices of futures with expirations T1 and T2:

$$\mathbf{F_{tT1} - F_{tT2} = BXM_t * (r_{tT1} - r_{tT2})}$$

## Performance of BXM Futures

### Daily Returns

The daily rate of change of BXM futures is driven first by the daily rate of change of the BXM, and second by the daily rate of change of the money market rate. The BXM leverages the total return of the S&P 500 in exchange for a short exposure to an S&P 500 call. The return on the S&P 500 call in turn moves with the S&P 500 and depends on its volatility. To gauge the net daily effect, we can assume a Black-Scholes setting and approximate the daily rate of return of the BXM by its instantaneous return. This approximation shows that day-to-day, an investment in the BXM provides:

(1) An exposure to the S&P 500 price return with a β coefficient equal to the leverage of the S&P 500 in the BXM portfolio times one minus the delta of the S&P 500 call. The leverage or weight of the S&P 500 in the BXM portfolio is equal to 1 plus the absolute weight of the call in that portfolio. The leverage increases with the price of the call.

(2) A leveraged exposure to the dividend yield of the S&P 500

[1[1]] The daily dollar variation of BXM futures is $\$100 * (F_t - F_{t-1}) = \$100 * (BXM_t * (1+r_{tT}) - BXM_{t-1} * (1+r_{t-1T}))$. The money market rate can be factored out if its daily change is relatively negligible.

(3) An exposure to the gamma of the call.

(4) A volatility proportional to the volatility of the S&P 500 by a coefficient equal to the leverage of the S&P 500 call times one minus the delta of the call.

Formally, the BXM expected rate of return and volatility are:

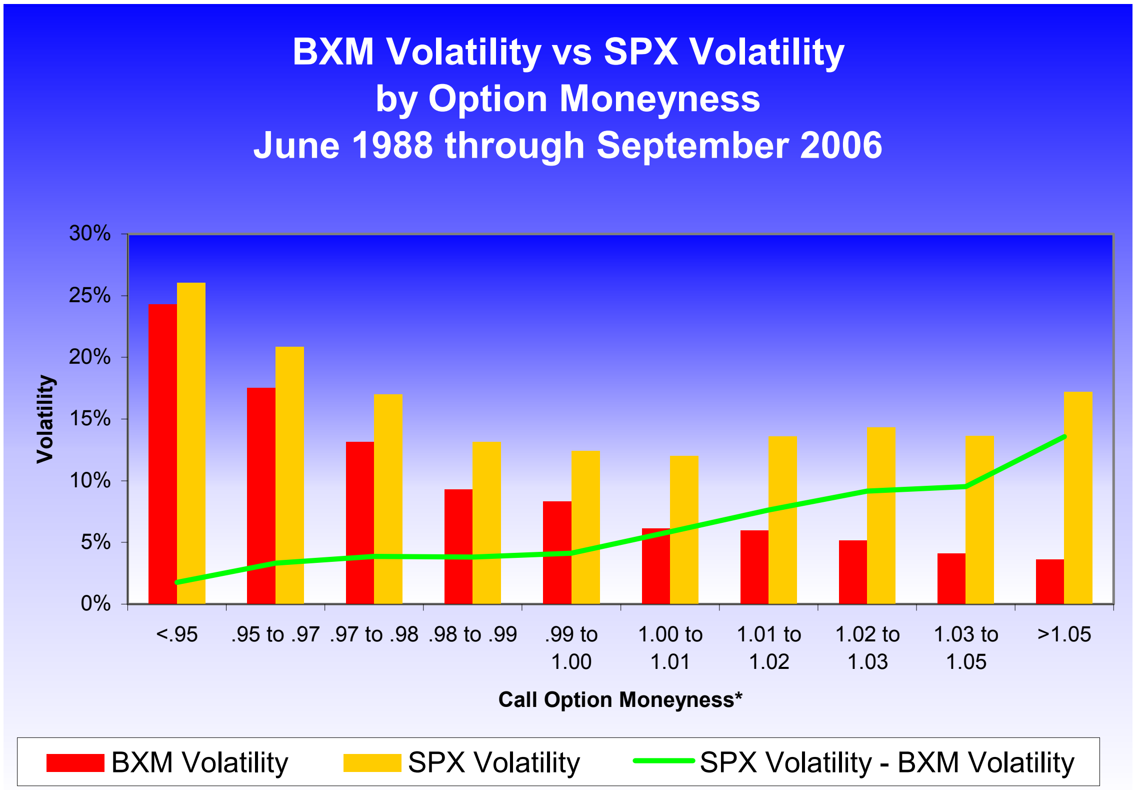
$$\mathbf{E[1 + R_t] \approx (1+ w_{t-1}) * [\,(1- \delta) * (1 + R^s) + d_t - \gamma\, S_{t-1}\, \sigma^2 / 2)\,]}$$

$$\boldsymbol{\sigma_{BXM} \approx (1 + w_{t-1})\,(1-\delta)\,\sigma}$$

where $w_{t-1} = C_{t-1} / (S_{t-1} - C_{t-1})$ is the absolute weight of the call in the BXM portfolio and $(1+ w_{t-1})$ is the weight or leverage of the S&P 500 obtained by selling the call; $S_{t-1}$ and $C_{t-1}$ are the values of the S&P 500 and call at the previous close, $R_s$ is the price return of the S&P 500 and d its dividend yield. The greeks σ, δ and γ are respectively, the volatility of the call, its delta and its gamma. The delta is the sensitivity of the call price to the S&P 500 and the gamma is the sensitivity of this sensitivity. To lighten notation the time indexes of the greeks are omitted from the formulas.

These equations show that the absolute weight and delta of the call are important determinants of the daily expected return and volatility of the BXM. From June 1988 to September 2006, the absolute weight of the BXM call has averaged around 1.6%, yielding a leverage factor for the S&P 500 of approximately 1.016. About 42% of observations have been below 1%, 72% below 2%, 96% below 5% and 99% below 7%. The weight has reached a maximum of approximately 13.5%, or a leverage factor of 1.135 for the S&P 500.

At the roll date when a new at-the-money call is sold, its delta is close to ½. Based on the historical record, the average beta of the BXM relative to the S&P 500 is then approximately ½ and the volatility of the BXM is approximately half that of the S&P 500. As the call moves in-the-money, its delta starts to converge to 1, and its volatility decreases, converging to 0 in the limit. If the call goes out-of-the-money the delta decreases, the volatility of the BXM increases, converging to 1 in the limit.




Turning to BXM futures, if the BXM and money market rates of return have approximately lognormal distributions, the mean log return of BXM futures is the sum of the mean log returns of the BXM and the money market rate. If the BXM daily rate is also independent of the money market rates, as it appears to be, then the volatility of the daily rate of change of BXM futures is equal to the square root of the sum of the variance of the BXM and of the relevant money-market rate. The statistics in the table below are calculated from 1, 3, and 6 months Eurodollar rates (EDM1, EDM3 and EDM6)

| Daily Log Returns | BXM | EDM1 | EDM3 | EDM6 |
|---|---|---|---|---|
| Mean | 0.0089 | -0.0002 | -0.0001 | -0.0001 |
| Annualized Volatility | 0.11 | 0.18 | 0.17 | 0.19 |
| Correlation | | -0.013 | -0.012 | 0.015 |
| BXM Futures Daily Log Returns Rough Estimates | | 1-Month | 2-Month | 3-Month |
| Mean | | 0.0086 | 0.0088 | 0.0087 |
| Annualized Volatility | | 0.21 | 0.20 | 0.22 |

**Roll-to-Roll Returns**

The return to holding a long BXM futures position from a BXM roll date to the next depends on whether the S&P 500 finishes out-of or in-the money.

(1) If the S&P 500 call finishes out-of-the money, the futures return per dollar of exposure is the difference between (a) the leveraged total gross return of the S&P 500 and (b) the money market rate between roll dates.

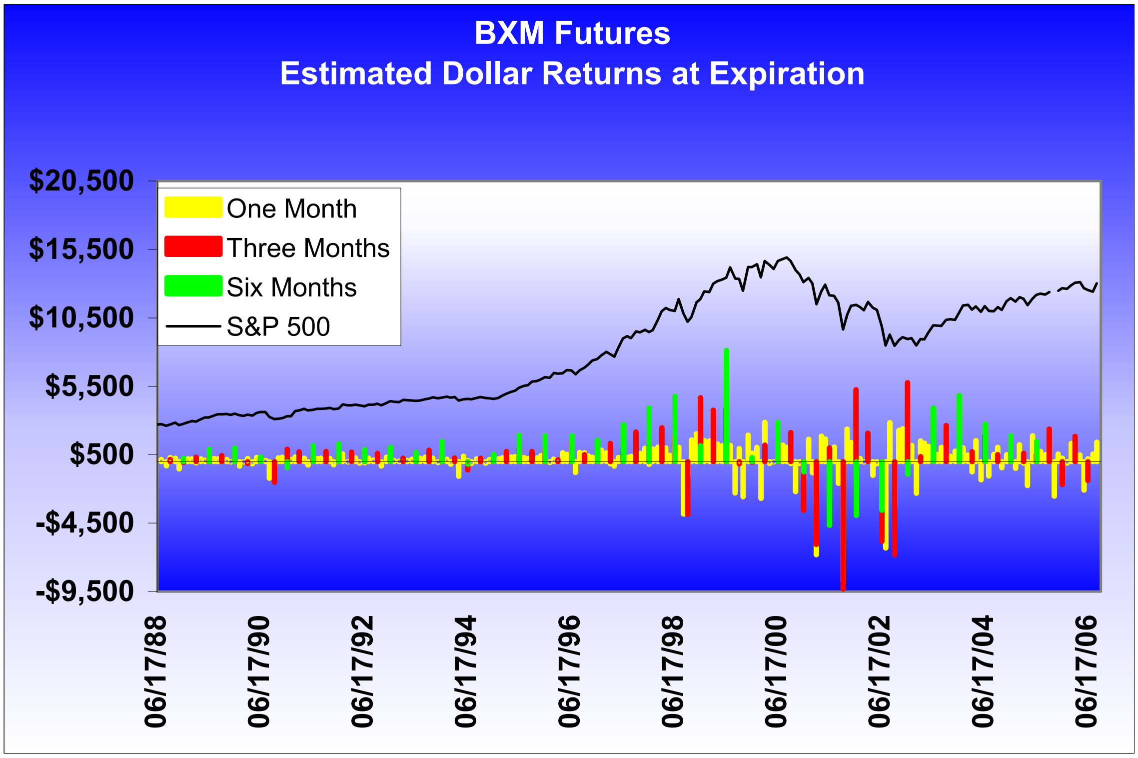
$$\$100 * BXM_{T-1} * [(1+w_{T-1})*(1+R_s) - (1+r_{T-1,T})]$$

(2) If the S&P 500 call finishes in-the-money, the futures dollar return per dollar of exposure is the difference between (a) the leveraged S&P 500 dividend yield plus the moneyness of the call and (b) the money market rate between roll dates. If the call is sold exactly at the money, this reduces to the difference between the leveraged dividend yield and the money market rate. The fact that the call is usually sold slightly out-of-the-money provides a cushion against S&P 500 downturns between roll dates. The trade-off is less leverage as the call price is smaller.

$$\$100 * BXM_{T-1} * [(1+w_{T-1})\ (d_{T-1,T} + m_{T-1}) - (1+ r_{T-1,T})]$$

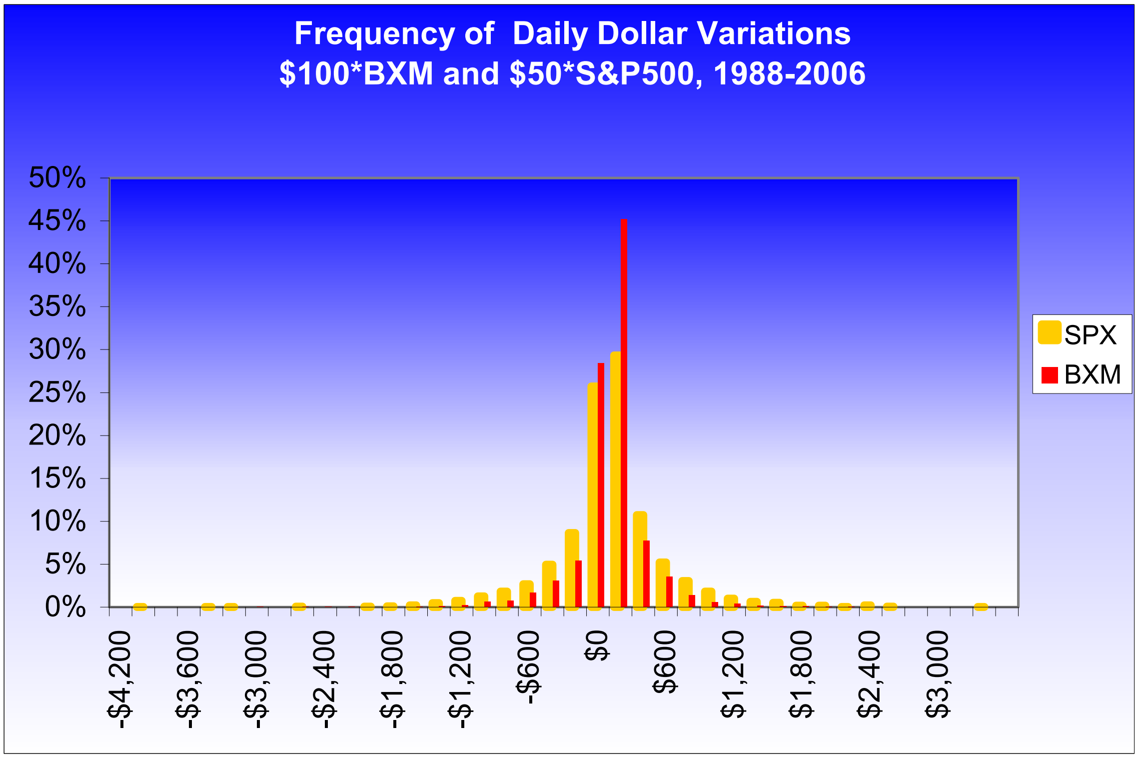
In the equations above, the size of the initial futures exposure is \$100 * $BXM_{T-1}$, the money market rate is $r_{T-1,T}$ $w_{T-1}$ is the absolute weight of the call when it is sold, $d_{T-1,T}$ is the S&P 500 dividend yield between roll dates, $m_{T-1}$ is the initial moneyness of the BXM call defined as $m = K_{T-1} / S_{T-1}$ , and $K_{T-1}$ is the strike price of the call sold at date T-1.

From June 1988 to September 2006, the moneyness of the call at the roll date has averaged 1.01 and its maximum was 1.02. The absolute weight of the call in the S&P 500 portfolio has ranged from .48% to 4.6% and has averaged 1.67%. The call has settled in-the-money 55.45% of the time. The next chart shows the estimated BXM futures dollar returns derived from this performance and that of the money market rate.




## BXM Futures versus S&P 500 Futures

Since both BXM and S&P 500 futures closely track their underlying index, the distribution of their daily index returns is a good proxy for the daily dollar variation of the futures. To simulate BXM futures, BXM returns are multiplied by $100 and to simulate S&P500 e-mini futures, S&P 500 returns are multiplied by $50. Based on data from 1988 to 2006, the distribution of daily dollar BXM futures is likely to be higher peaked and to have thinner tails than the distribution of daily dollar returns of S&P500 e-mini futures.

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