Over the past several years, equity-index volatility products have emerged as an asset class in their own right. In particular, the use of variance swaps has skyrocketed in that time frame. A recent estimate from Risk magazine placed the daily volume in variance swaps on the major equity-indices to be US$5m vega (or dollar volatility risk per percentage point change in volatility). Furthermore, variance trading has roughly doubled every year for the past few years.

Along with the proliferation of the breadth and complexity of available volatility products has come increased anxiety and confusion about how investors can most effectively and efficiently trade volatility. We offer a brief overview of the concept of variance and volatility; describe how a variance swap can be used to trade equity-index volatility; and illustrate some advantages that variance swaps offer over other volatility-based assets. Lastly, we will describe how CBOE variance futures contracts are essentially the same as an OTC variance swap.

Volatility and variance

Volatility and variance are measures of the level of variation of an asset's price over time. Even though volatility is the more commonly used term in the financial markets and media, an asset’s volatility is actually derived from its variance, as we will see below. An asset with high volatility is expected to move around more, in percentage terms, than a low-volatility asset.

A single asset can also have a period of low volatility followed by a period of high volatility and vice-versa. For example, the S&P 500 Index saw some periods of historically low volatility in the years leading up to 2007, but its volatility increased dramatically in the summer of that same year. Investors can use variance swaps and other volatility-based products to hedge against or speculate on the differences in volatility across products and time.

Definition of variance and volatility

In mathematical terms, the annualised variance of an asset can be expressed as follows:

$$\text{Variance} = 252 \times \left( \sum_{i=1}^{N} R_i^2 / (N-1) \right),$$

where $R_i = \ln \left( P_{i+1} / P_i \right)$ is the percentage return of the asset from day $i$ to day $i+1$ and $N$ is the number of prices observed. Note that this is a standard textbook definition of variance under the assumptions that (i) there are 252 trading days in a year and (ii) the average daily asset return is 0. The latter assumption is a convenience that is typically referred to as the zero-mean assumption and has a relatively minor impact on the calculation assuming that an asset’s variance is sufficiently high. The volatility of an asset can then be expressed as the square root of its variance.
Assets with varying volatilities

Exhibit 1

S&P 500 Index one-month volatility

Exhibit 2

Source: Chicago Trading Company, LLC
Realised versus implied variance and volatility
The terms variance and volatility as we have defined them could more technically be referred to as realised variance and volatility. However, some financial markets may imply a variance going forward that differs from what has occurred recently. The market’s expectation of variance going forward is referred to as implied variance. Exhibit 3 demonstrates how implied volatility closely tracks historical volatility, but that the implied calculation is more reactive to anticipated changes in volatility. As we will see later on, the distinction between realised and implied variance is an important component of understanding how variance swaps and variance futures are priced in the marketplace.

What is a variance swap?
A variance swap is not really a swap in the traditional sense. The term swap typically refers to a structured contract consisting of periodic cash flow exchanges (usually in the fixed income or foreign exchange markets). Variance swaps are in fact forward contracts with a payoff based on the realised variance of a stated equity index. More precisely, the payoff of a variance swap is given by the formula:

\[ \text{Settlement} = \text{Notional} \times (\text{Realised Variance} - \text{Variance Strike}), \]

where realised variance is defined as above. The variance strike is a fixed number that reflects the trade price and the market’s expectation of realised variance at the time that the variance swap is entered. The variance strike is often quoted as the square root of variance (e.g., a 240.25 variance strike would be denoted by \( \sqrt{240.25} \)) to allow investors to easily relate the quantity back to volatility terms.

A variance swap allows the buyer and seller to gain exposure to changes in the variance of the underlying index. Market participants can trade a variance swap to hedge off exposure from other areas of their businesses or to profit from anticipated changes in the variance of an asset. We will explore other possible uses of variance swaps in more detail below.

An alternative to a variance swap is a volatility swap, which has a payoff based on the realised volatility:

30-day historical versus implied volatility

Exhibit 3
Settlement = Notional \times (Realised\ Volatility – Volatility\ Strike).

However, we will demonstrate below that volatility swaps have several properties that make them less optimal than variance swaps for trading variance.

Uses of variance swaps and futures

In principle, any institution which seeks to hedge or speculate on volatility might want to strongly consider trading variance, either in the form of an OTC variance swap or a CBOE variance future. Like other derivatives, variance swaps are employed by many to hedge risk. In particular, some businesses have natural exposure to volatility that could be reduced by trading variance swaps.

One of the more intriguing uses proposed for variance swaps is as a diversification instrument for a long-only equity portfolio. The presumption is that as equity markets fall volatility tends to rise. Going long a variance swap can provide an offset for a long-only fund in falling market conditions. If one believes that volatility is negatively correlated with the directional movements in the broader stock market, a variance swap, it could be argued, would possess potentially valuable diversification characteristics.

Other potential users of variance contracts as a hedging vehicle include:

- Investors seeking to hedge against decreased liquidity since liquidity tends to decrease during increased levels of volatility.
- Convertible bond funds, which are typically long corporate convertible bonds and short corporate equities. These funds are naturally long volatility, so they might use variance swaps as a hedge against a fall in volatility.
- Insurance companies that might like to hedge some of their underlying business exposures to volatility in the marketplace.
- Option trading firms which warehouse significant volatility risk (long or short) may want to use variance swaps to expediently offset their exposure to market fluctuations. Additionally, option trading firms typically benefit from higher volatility since it correlates to increased trading activity.

Market participants might also use variance swaps to gain access to exposures which are viewed as potentially profitable trading or investment opportunities. Variance swaps allow position-taking in ‘pure’ volatility, independent of the other risks that would accompany an option-based volatility strategy. Potential volatility trading strategies include:

- Trading a (long) variance swap on one index or asset versus a (short) variance swap on another index or underlying.
- Entering into other relative value trades (e.g., buying a one-year variance swap and selling a nine-month variance swap, which is effectively a play on the expected three-month variance or volatility in nine months’ time).
- Trading variance swaps on an index versus variance swaps on the individual components of that index (a dispersion or correlation trade).

Advantages of variance swaps and variance futures

Around 80% of market participants who assert that they trade volatility do so by trading options (or embedded options, as is the case with convertible bond funds). A common strategy is to trade options on a delta-neutral basis, meaning that the trader will hedge the exposure of the option to the underlying in order to isolate the exposure to volatility. The disadvantage of this approach is that returns will hinge not only on market volatility but also on the cost of constantly re-hedging a portfolio to eliminate directional risk. Additionally, an option’s sensitivity to volatility will diminish as the underlying moves away from its strike price. Therefore, large market moves can cause an option to become a purely directional play on the underlying. Trading variance eliminates the need to constantly re-hedge or rebalance the market directional risk as the market moves about.

Variance swaps are, in some sense, a ‘natural’ product to trade, given a view or a concern about volatility over time. In the Black-Scholes option valuation formula, every time one sees a term representing the annualised volatility (or standard deviation) of an underlying asset’s return, it is multiplied by the square root of time. Every time one sees
a term representing the annualised volatility squared (or variance), it is scaled by time. The fact that variance is linear (or additive) in time means that a variance swap is relatively easy to value even after it is initially traded. A seasoned variance swap that has begun to accrue realised variance can be unwound by doing an offsetting variance swap trade for the remaining life of the initial contract.

Volatility swaps are not so accommodating; because of their dependence on the square root of time, they are less easily valued and unwound. Seasoned volatility swaps ‘retain their history’ in a way that seasoned variance swaps do not. Another reason for the popularity of variance swaps is that they may be hedged in a static way under a broad range of circumstances using a portfolio of options.

While variance swaps and volatility swaps both give the investor volatility exposure, the fact that variance swaps are easily valued and have static hedges make them the current variance and volatility product of choice. It is estimated that variance swaps (as opposed to volatility swaps) constitute well over 90% of the over-the-counter (OTC) market.

**Variance swap markets**

Currently, the greatest percentage of variance swap trading on the S&P 500 Index takes place in the OTC market. As we will show below, however, the CBOE variance futures contract offers an alternate vehicle for effectively trading the same thing. Additionally, the CBOE contracts offer the added advantages of efficient price discovery and elimination of cross-party risk. “CBOE variance futures market participants benefit from the transparency of robust two-sided markets posted by the CBOE Futures Exchange specialist,” according to Jay Caauwe, Business Director of the CBOE Futures Exchange, the wholly owned Futures Exchange for the CBOE. Caauwe further notes that “the contracts have the guarantee of efficient clearing through the triple-A rated Options Clearing Corporation (OCC).”

**OTC market conventions**

As mentioned above, the settlement formula for an OTC variance swap is given by:

\[
\text{Settlement} = \text{Notional} \times (\text{Realised Variance} - \text{Variance Strike})
\]

The exact formula for realised variance is given by:

\[
\text{Realised Variance} = 252 \times \left( \frac{\sum_{i=1}^{N} R_{i}^2 / N_{e}}{N_{e}} \right) \times 100\% ,
\]

where \(N_e\) is the actual number of days in the observation period and \(N_e\) is the expected number of days in the period. The actual and expected number of days can differ if a market disruption event occurs. For example, most US markets were closed on June 11, 2004 to mark the passing of former president Ronald Reagan. The closing affected the number of observations contributing to the realised variance calculation \((N_e)\), but it did not affect the denominator \((N_e)\).

One possible source of confusion when comparing the definition above to our definition of realised variance at the beginning of this article is that in our original definition we divide by \(N-1\) and not \(N\). The resolution lies in the fact that the denominator in our original definition refers to the number of prices whereas the denominator above refers to the number of percentage returns, or yields. Since there will always be one less yield than the number of prices, these definitions are the same. While this is a minor technical point, we will see below that it is an important step in

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understanding the connection between OTC variance swaps and CBOE variance futures.

As a matter of convention, the notional amount of a variance swap is scaled so that if the realised volatility moves by one point, the payoff of the variance swap moves approximately by an agreed upon vega notional amount. Since variance is the square of volatility, the rate of change of variance is twice that of volatility, which motivates the relationship:

\[
Variance\,\,Notional = \frac{Vega\,\,Notional}{2 \times \sqrt{\text{Variance\,\,Strike}}}.
\]

For example, if the variance strike is 16\(^2\) and the vega notional is 100,000, then the variance notional is \(\frac{100,000}{2 \times 16}\) = 3,125. If the realised variance for the contract period is 17, then settlement value of the contract is:

\[
3,125 \times (17^2 - 16^2) = 103,125.
\]

**CBOE variance futures**

The CBOE variance futures contracts offer an alternative to variance swaps. They provide an opportunity to gain the same exposure to variance as their OTC counterpart. These products trade on the CBOE Futures Exchange with quarterly expirations and are listed under the futures symbols VT (for three-month variance) and VA (for 12-month variance). Per its contract specifications, the price of a CBOE variance futures contract at maturity is:

\[
\text{Realised Variance} = 252 \times \left( \sum_{i=1}^{N_e-1} R_i^2 / (N_e-1) \right) \times 100\%.
\]

As mentioned above, this definition is in fact identical to the settlement value for a variance swap under the realisation that \(N\) prices map to \(N-1\) yields.

The contract multiplier for the CBOE variance future contracts is US$50 per futures point change. Thus, at the beginning of the realised variance observation period, trading a single variance futures contract is equivalent to trading a US$50 variance notional variance swap. This identity does not hold once the contract enters its observation period, because the denominator of the futures contract remains fixed throughout the period. For example, trading a 12-month variance contract half way through its observation period is equivalent to trading a US$25 variance notional with six months to expiration.

**Replicating a variance swap with a variance future**

In order to replicate a variance swap with a variance future having the same expiration date, we calculate the variance notional and variance strike implied by the variance future price. We first observe that once a variance future has entered its observation period, its price can be decomposed as:

\[
\text{Variance\,Future\,Price} = \frac{(M-1) \times \text{RUG} + (N_e - M) \times \text{IUG}}{N_e - 1},
\]

where \(M\) is the number of prices observed to date, \(\text{RUG}\) is the realised variance to date, and \(\text{IUG}\) is the market implied variance strike for the time remaining until the contract expires. In words, the variance futures price is a weighted average of the realised variance to date and the implied variance remaining for the observation period. The realised variance is weighted by the number of observations that have occurred, and the implied variance is weighted by the number of observations that remain.

Given the above formula, it is easy to compute the variance notional and variance strike from the variance future price:

\[
\text{Variance\,Notional\,of\,Future} = 50 \times \frac{N_e - M}{N_e - 1},
\]

\[
\text{Variance\,Strike} = \text{IUG} = \frac{(N_e - 1) \times \text{Future\,Price} - (M - 1) \times \text{RUG}}{N_e - M}.
\]

To illustrate these calculations, we will go through a detailed example using the 12-month variance future expiring on December 21, 2007 (VAZ7) at the end of the trading day on February 23, 2007:

- The observation period for this contract started on December 15, 2006, and ended on December 21, 2007, so \(N_e = 257\) for the period.
- As of the day of our example, there were \(M = 46\) prices that had been observed. The \(\text{RUG}\) on that day was 47.43 or 6.89\(^2\). (The \(\text{RUG}\) for active 12-month variance futures is available on Bloomberg as symbol RIK for March...
The future closed at 153.00 or 12.37°.

Using these inputs, the variance strike can be computed:

\[
\text{Variance Strike} = \left(153.00 - \frac{47.43}{251} \times 257 - 46\right) \times \frac{257 - 46}{257 - 1} = 175.51 \times 13.25°.
\]

The variance notional per future is \(50 \times \frac{257 - 46}{257 - 1} = 41.02\) .

If we want to replicate a variance swap with the same expiration and 100,000 vega notional, we first observe that the vega notional of the same swap is \(\frac{100,000}{2 \times 13.25} = 377410\) .

Thus, we would trade \(\frac{377410}{41.02} = 92\) contracts to replicate the variance swap.

### Stripping variance futures

One of the primary advantages that variance swaps offer over other volatility-based products is that variance is additive, meaning that multiple variance futures contracts can be stripped together to create a single long-term contract. The mechanics of stripping together futures contracts can best be illustrated by an example.

Suppose in the previous example that instead of trading a variance swap that expires on December 21, 2007, we want to trade a contract that expires on December 19, 2008. We can do so by combining a position in both the 12/07 (VAZ7) and 12/08 (VAZ8) futures contracts. The key is to recognise that the VAZ8 contract expires into the realised variance from December 21, 2007 to December 19, 2008. If we return to February 23, 2007, as in our previous example:

- The VAZ7 contract had 257-46 = 211 observations remaining.
- The VAZ8 contract had all of its 252-1=251 observations remaining.
- The closing price of VAZ8 on the example date was 253.50 or 15.92°.

We can now calculate the variance notional and strike of the stripped contracts as the weighted average of the individual contracts:

\[
\text{Variance Notional} = \frac{211 \times 41.02 + 251 \times 50}{211 + 251} = 45.90
\]

\[
\text{Variance Strike} = \frac{211 \times 175.51 + 251 \times 253.50}{211 + 251} = 217.88 = 14.76°
\]

So, if we want to replicate a variance swap with 100,000 vega notional, we observe that the desired variance notional of the strip is \(\frac{100,000}{2 \times 14.76} = 3387.53\) . Thus, we would trade \(\frac{3387.53}{45.90} = 73.81\) of the strips to replicate the variance swap, which translates into \(73.81 \times \frac{211}{211 + 251} = 34\) of the VAZ7 contracts and \(73.81 \times \frac{251}{211 + 251} = 40\) of the VAZ8 contracts.

### Commonly asked questions about CBOE variance futures

**Q:** What term structures do CBOE variance futures offer?

**A:** CBOE variance futures offer the opportunity to trade from the current trade date to any of the nearest four quarterly expirations, plus the next two Decembers.

**Q:** If I trade a CBOE variance future in between two quarters, doesn’t the ‘realised’ portion of the contract make it different compared to a true spot-starting variance swap?

**A:** Not at all. They will perform the same. The only thing you need to keep in mind is that an additional step will be required to determine the true volatility level and number of contracts that you will be trading (see above examples).

**Q:** What kind of market widths can I expect to see with CBOE variance futures?

**A:** This will vary, as there are no set requirements, but you can typically see markets that are about one half of a volatility point wide.

### Conclusions

The explosive growth of volatility-based products in recent years clearly reflects a demand for a traded vehicle which can be used to hedge or to implement a view on volatility. The user base for these products continues to expand from...
sophisticated trading firms and hedge funds to insurance companies, risk managers, and fundamental investors. To date, OTC variance swaps have accounted for the majority of trading in this field, but, as is shown in this article, CBOE variance futures contracts can generate the same volatility exposures as OTC variance swaps with the additional benefits associated with exchange-traded products.

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